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Fractional quantization of ballistic conductance in 1D electron and hole systems

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Abstract
In the present paper we give a review of our recent results in the field of the '0.7 anomaly' and related phenomena in 1D electron and hole systems. We introduce the concept of a fractional quantization of the ballistic conductance arising from exchange interaction of the Heisenberg type between the carrier localized in the region of the quantum point contact and freely propagating carriers and show that the conductance pattern is qualitatively different for electron and hole systems.

1. Introduction

Mesoscopic physics is now one of the most rapidly developing branches of condensed matter physics. Due to the advances in nanotechnology the experimental realization of quasi-one-dimensional channels became possible about two decades ago. In these objects the motion of the carrier (electron or hole) is confined in two dimensions and is free in the third one. Depending on the ratio between the length of the channel and the inelastic scattering length, the transport of the carriers in these objects can be either of a diffusive or ballistic nature. In the latter case all inelastic processes accompanied by Joule losses take place in 2D reservoirs connected with a channel but not in the channel itself. In this limit the conductance of the sample is proportional to a conductance quantum $G_0 = 2e^2/h$ [1, 2],

$$G = 2\frac{e^2}{h}N, \tag{1}$$

where N is the number of open propagating modes. It can be changed by tuning of the side gate voltage $V_{\rm g}$, which allows experimental observation of the ballistic conductance staircase [3, 4].

However, in the region of the small electron concentrations when only one propagating mode is open, the experimentally measured ballistic conductance qualitatively deviates from (1). Namely, a mysterious additional plateau regularly appears around $G \approx 0.7G_0$ [5–7].

Although being formally analogical to the plateaux appearing in the fractional quantum Hall effect [8], the physical origin of this '0.7 anomaly' is of course very different. Two experimental facts indicate that it is somehow connected with spin. First, it was shown that decrease of the concentration results in a drastic increase of the effective electron g-factor [5]. Second, the application of the external magnetic field leads to the smooth evolution of the value of the conductance on the additional plateau from about $0.7G_0$ to $0.5G_0$. The latter value is indeed expected for the case of Zeeman splitting of the propagating mode.

Later experimental study revealed that fractional quantization of ballistic conductance is not necessarily universal. It seems that in long quantum wires the value of the conductance on the additional plateau is closer to $0.5G_0$ than to $0.7G_0$ [9]. It is also probable that situation can be qualitatively different in the samples with electron and hole types of conductivity. In the latter case the experimental data existing to date seem to be rather controversial. The first experimental findings of the fractional quantization of the ballistic conductance for the holes indicated that it varies from sample to sample and depends strongly on the offset between the bands of the light and heavy holes [10]. Other groups reported findings of a fractional plateau in p-type systems in the same position as it was found for electrons [11, 12]. The situation now is thus very far from being clear.

In this review we address theoretically a complex set of the phenomena related to the '0.7 anomaly' and its analogs in ballistic 1D samples with p- and n-types of conductivity. The paper is organized as follows. In section 2 we present the theoretical overview of the various theoretical models. In section 3 we introduce the concept of fractional quantization of the ballistic conductance arising from the exchange interaction of the Heisenberg type between localized and propagating electrons. In section 4 we consider the case of the holes and show that it can be qualitatively different from the case of the electrons. The conclusions summarize the main results of the work.

Theoretical overview

Since its first experimental observation the 0.7 anomaly has represented a challenge for the theorists working in the field of condensed matter physics. The proposed scenario of its appearance varied from Wigner lattice formation [13] and non-Fermi liquid behavior of the quasi-1D electron gas [14] to electron–phonon interactions [15]. However, although each of these models was describing some particular aspect of the phenomenon, a theory which gives a self-consistent explanation of all relevant experimental facts still does not exist.

In the present section we give a short overview of the existing theoretical results which seem most promising to us and which are related to our concept of a fractional quantization of ballistic conductance resulting from the exchange interaction between net spin localized in the region of a QPC and freely propagating carriers [16].

We start with phenomenological models which attribute the appearance of the additional plateau to the formation of the gap between two spin bands in the region of the QPC. Initially proposed in the work of Bruus *et al* [17], this concept appeared to be rather promising in phenomenological description of the phenomenon and permitted to obtain simple closed expressions for the conductance as a function of the chemical potential and temperature. The model of Reilly *et al* [18–20] is the most elaborate of the class of the models described above. Contrary to [17], Reilly assumes that a spin gap is not a constant, but depends on the side gate voltage V_{sg} linearly. The model operates with two parameters. The first one is the capacitance of the QPC, which governs the dependence of the Fermi energy of a 1D electron gas on V_g, and the second one is the rate of the opening of the spin gap with V_{sg}, $\gamma = \mathrm{d}\Delta E_{\uparrow\downarrow}/\mathrm{d}V_{sg}$. Depending on these parameters various scenarios for the 0.7 anomaly can be accounted for. At low temperatures the model predicts the appearance of the additional plateau at $0.5G_0$ associated with a fully spin polarized propagation mode. The standard value of the conductance $2e^2/h$ is recuperated when a decrease of the side gate voltage puts the edge of the other spin subband below the Fermi level of the 2DEG. The increase of the temperature leads to the increase of the conductance at the additional plateau due to the contribution of thermally excited electrons from the opposite spin band to the current, which remains almost constant in a small region of V_g due to the continued opening of the gap. In the case when a spin gap $\Delta E_{\uparrow\downarrow}$ is much smaller than the thermal excitation energy $k_B T$ the additional plateau disappears.

A similar result was earlier obtained by one of us [21] within a phenomenological model of a quantum wire with partially polarized electron gas. The momentum dependence of the state occupancy was approximated by a step function (f is a Fermi function) $n(q,T) = [1 + f(T, \varepsilon(q) - \mu + \Delta E_{\uparrow\downarrow})] f(T, \varepsilon(q) - \mu)$ with chemical potential μ determined by 1D carrier concentration n_{1D}. Assuming that in some range of n_{1D} one has $\mu(n_{1D}) - \Delta E_{\uparrow\downarrow}(n_{1D}) = -\xi = \text{const}$ and restricting consideration to the case of moderate temperatures $\mu/kT \gg 1$, the additional conductance plateau appears at

$$G = \frac{e^2}{h}\left\{1 + \frac{1}{\exp(\xi/kT) + 1}\right\}. \tag{2}$$

An increase of the temperature leads to an increase of the conductance from $G = (1/2)G_0$ to $G = (3/4)G_0$, which qualitatively corresponds to a stable observation of the unified 0.7 feature.

The origin of the spin gap can be attributed to spontaneous spin polarization due to the exchange interaction in the regime of the small carrier concentration. Qualitatively, the effect can be interpreted as follows. The linear density of the kinetic energy of 1D electron gas density is proportional to the cube of the linear concentration n_{1D}^3 and reads

$$\varepsilon_{kin}^{1D} = \frac{\pi^2\hbar^2 n_{1D}^3}{6mg_s^2}, \tag{3}$$

where g_s is the spin factor giving the number of electrons per unit cell of phase space. Naturally, it is minimal for the unpolarized gas with $g_s = 2$. On the other hand, the exchange energy can be estimated as being proportional to $\epsilon_{exc} \sim -n_{1D}^2/g_s$ and thus favors spin polarization. The competition between two terms results in spontaneous polarization in the region of small concentrations, where the term quadratic in concentration dominates over the cubic one, and depolarization in the opposite limit. This corresponds to the well known Stoner ferromagnetic instability, whose existence was recently reported experimentally for 2D systems [22].

Using the Hartree–Fock approximation the exchange interaction energy per unit length of the quasi-1D electron gas can be estimated as [23]

$$\begin{aligned}\epsilon_{exc} &= -\frac{1}{2L}\sum_{K,Q<K_F}\langle KQ|V|QK\rangle \\ &\approx \frac{0.28e^2}{g_s}n_{1D}^2 + \frac{e^2}{4g_s}n_{1D}^2\ln\left(\frac{n_{1D}R}{\pi g_s}\right),\end{aligned} \tag{4}$$

where R and L are the radius of the wire and its length respectively. Together with (3), this allows to estimate a spin gap as

$$\Delta E_{\uparrow\downarrow} \approx 2n_{1D}e^2\left[0.15 - 0.25\ln\left(\frac{n_{1D}R}{\pi}\right)\right] - \frac{\pi^2\hbar^2 n_{1D}^2}{2m}. \tag{5}$$

The linear concentration can be estimated as a function of side gate voltage and capacitance c between the gap and 1D electrodes, $n_{1D} = cV_{sg}/e$. The first term in (5) describes the continuous opening of the gap in the region of small concentrations where exchange interaction dominates. It is accounted for in Reilly's model and allows the theoretical estimation of his phenomenological parameter γ. On the contrary, the second term describing the quenching of the spin gate coming from kinetic energy was neglected.

The exchange-interaction-induced spin gap in infinite quasi-1D wires was also considered in the numerical density-functional calculations of Wang and Berggren [24]. In accordance with qualitative analysis presented above, they reported a large subband splitting and full spin polarization at low electron densities. At the same time it was emphasized that the effect can be partially suppressed if correlation effects were taken into account (the same is also true for the Hartree–Fock description considered above).

All phenomenological models have the following two shortcomings. First, they predict that the value of the conductance at the split-off plateau is non-universal and can lie anywhere between $0.5G_0$ and G_0, depending on the parameters used. Second, the conductance on the additional plateau decreases with temperature and reaches $0.5G_0$ when $T \to 0$, which contradicts the existing experimental data.

To address the experimentally observable temperature dependence of the 0.7 feature, Yigal Meir and coworkers proposed a scenario based on many-body Kondo physics [25]. Indeed, the experiments of the Harvard group [26] indicated certain similarities between conductances in quantum dots (QDs) in the Kondo regime and in QPCs in the regime of the '0.7 anomaly', namely:

(i) Formation of a sharp conductance peak for the low temperature regime centered at the Fermi level (zero bias anomaly, ZBA).
(ii) Universal single parameter scaling of the conductance with temperature in a wide range of the side gate voltages with scaling parameter conveniently labeled as the system's Kondo temperature T_K,

$$G = 2e^2/h[1/2f(T/T_K) + 1/2]. \quad (6)$$

The universal function $f(T/T_K)$ coincides with those for QDs in the Kondo regime [27, 28], the only difference between the two systems being the factors of 1/2 in (6). It satisfies the following boundary conditions: $f(0) = 1$ and $f(\infty) = 0$.
(iii) The Zeeman splitting of the zero bias peak under a strong magnetic field.

Inspired by the experimental observation (ii), which gives a universal conductance value close to e^2/h in the high temperature limit, Yigal Meir and coworkers proposed to describe a QPC in the regime of the '0.7 anomaly' by means of the following model Hamiltonian:

$$H = \sum_{k\sigma\in L,R} \epsilon_k c^\dagger_{k\sigma} c_{k\sigma} + \sum_\sigma \epsilon_{d\sigma} d^\dagger_\sigma d_\sigma + U n_{d\uparrow} n_{d\downarrow} + \sum_{k\sigma} [V_k^{(1)}(1 - n_{d\bar\sigma}) c^\dagger_{k\sigma} d_\sigma + V_k^{(2)} n_{d\bar\sigma} c^\dagger_{k\sigma} d_\sigma + \text{h.c}], \quad (7)$$

where $c_{k\sigma}$ correspond to the annihilation of the electron with a spin σ in the leads and d_σ to the annihilation of the electron bound in QPC region. The couplings $V_k^{(2)}$, $V_k^{(1)}$ were treated as energy-dependent step-like functions, describing $0 \leftrightarrow 1$ and $1 \leftrightarrow 2$ charge fluctuations in the QPC respectively. Differently from the original Anderson Hamiltonian in (7), $V_k^{(2)} < V_k^{(1)}$, which reflects the fact that if the QPC contains a single electron Coulomb repulsion decreases the hybridization between the QPC and the leads as compared to the case of an empty QPC. It was argued that at high temperatures the Hamiltonian (7) should give the conductance $G = G_0/2$, arising from $0 \leftrightarrow 1$ fluctuations, while decrease of the temperature leads to the Kondo enhancement of the conductance due to $1 \leftrightarrow 2$ fluctuations until its standard value $2e^2/h$ is recovered at $T = 0$.

For the purposes of quantitative analysis by applying the Schrieffer–Wolff transformation [29] Hamiltonian (7) was mapped into a Kondo-type one with exchange coupling dependent on hybridization matrix elements $V_k^{(1,2)}$. These couplings were then treated perturbatively, which allowed the qualitative reproduction of the ZBA. At the same time, this approach fails to reproduce the standard value of the ballistic conductance at $V_{ds} \to 0$. We believe that this unitary limit could be further investigated through the Kubo linear response theory treating exactly the couplings between the bound state and the reservoirs. To our mind, the other shortcoming of the approach considered is that the height of the additional plateau is nonuniversal and strongly depends on the parameters of the model and a temperature. The model fails to explain qualitatively why at short QPCs the additional plateau is observed to be stable around $0.7G_0$ and not, say, $0.85G_0$.

It should be also noted that some new experimental results seem to contradict the main conclusions of a Kondo model. Namely, Graham and coauthors in their recent paper [30] reported the absence of ZBA for at least some experimental configurations. Another experiment done with QDs reported a stable observation of the anomaly down to extremely low temperature [31], where according to the predictions of the model considered above it should disappear. Concluding, the role of the Kondo correlations in the '0.7 anomaly' still remains an open question.

The stable observation of the conductance around $0.7G_0$ got its explanation in the models of the fractional quantization of the ballistic conductance, based on the interplay between singlet and triplet propagation channels for a pair of electrons. The first model of this type was initially proposed by Flambaum and Kuchiev [32], who considered the transport of the bound electron pairs through QPC. If exchange interaction splits the energies of singlet and triplet configurations, the effective potential barrier seen by a pair in the region of QPC becomes spin-dependent. If the energy of a triplet state is lower, there exists a region of the side gate voltages in which all triplet pairs can pass through QPC while all singlet pairs are reflected. As in the absence of the external magnetic field the probability of realization of a triplet state is 3/4 against 1/4 for the singlet one, the value of the conductance in this region should be $3G_0/4$. In other hypothetical case, when the energy of a singlet state lies below the energy of a triplet state, the situation is inverted and conductance reads $G = G_0/4$. If bound states containing more then one electron are formed, the existence of extra fractional plateaux was qualitatively predicted.

The weak point of the model of Flambaum and Kuchiev is that it operates with some hypothetical attraction between the electrons leading to a formation of a pair. Although such possibility cannot be completely dismissed *a priori*, the origin of this attraction rests unclear, which makes the scenario rather dubious. However, there exists a modification of the model based on the interplay of singlet and triplet scattering channels which does not assume the existence of the attraction between electrons. It was initially proposed by Rejec *et al* [33–35] and assume a formation of the bound single electron state in the region of the QPC[3] which interacts with propagating electrons via an exchange term of the Heisenberg type. If the energy of the traveling electron is small enough, one of the electrons always remains bound after the scattering process [37, 38], and transmission probabilities can be calculated using the Landauer–Buttiker formula

$$G_{T=0}(E_F) = \frac{2e^2}{h}\left[\frac{1}{4}T_s(E_F) + \frac{3}{4}T_t(E_F)\right], \tag{8}$$

where T_s and T_t are transmission coefficients for singlet and triplet configurations respectively and the coefficients 1/4 and 3/4 reflect their corresponding probabilities of realization. For finite temperatures, the generalization of the result (8) is straightforward and reads

$$G(T,\mu) = \int_0^\infty G_{T=0}(E)\left(-\frac{\partial f(T,E,\mu)}{\partial E}\right)\mathrm{d}E. \tag{9}$$

Rejec *et al* [35] claimed that the effective potential seen by the propagating electron consists of two barriers with a minimum at the center of the bulge. It contains resonant quasilevels for the singlet and triplet configurations, split by exchange interaction. The resonant tunneling through these levels corresponds, under certain condition, to the simultaneous appearance of fractional conductance plateaux with $G \sim (1/4)G_0$ and $G \sim (3/4)G_0$. On the other hand, for rectangular potential barrier only one plateau can be observed, depending on the sign of the exchange interaction constant [32, 39].

The many-electron correlations, that are similar to those of the Kondo type [40], lead to the temperature-dependent renormalization of the exchange interaction constant. According to the scaling argument of Anderson [41], in the case of the antiferromagnetic interaction it diverges for low temperatures, leading to the formation of a Kondo cloud around the localized magnetic moment. In the ferromagnetic case the exchange interaction constant goes to zero for $T \to 0$, thus leading to the quenching of the splitting between singlet and triplet states and disappearance of the '0.7 feature' at extremely small temperatures [39].

The possibility of the formation of the state with an unpaired spin in the region of the QPC was the focus of a number of theoretical works. The results obtained using numerical density-functional calculations support the hypothesis of the formation of either a net spin polarization in the region of the QPC arising from electron–electron interactions [42–45], which can serve as a dynamical spin filter, or a three-electron antiferromagnetic spin lattice inside the QPC [25, 46] acting as an effective spin one-half.

It should be noted that there exists experimental evidence that supports the scenario of single and triplet channels described above, coming from measurements of the nonequilibrium current noise in quantum wires and QPCs [47, 48]. In particular, measurements of the Fano factor which demonstrate the deviations from Poissonian noise were shown to be consistent with the ratio 3:1 for the triplet–singlet statistical weights [49].

We believe that these results allow treatment of the 0.7 anomaly as resulting from the exchange interaction between a propagating carrier and a magnetic moment J localized in the contact. Initially formulated in the works of Rejec *et al* for the case of a single localized electron with $J = 1/2$, this approach can be generalized for the cases of N localized electrons and holes. In these latter cases the height of the split-off plateau is a simple fraction of G_0 different from the value of $(3/4)G_0$ predicted in [32, 34]. This allowed us to introduce the concept of fractional quantization of the ballistic conductance considered in detail in the next two sections.

2. Fractional quantization of ballistic conductance in 1D electron systems

In this section we introduce the concept of fractional quantization of the ballistic conductance in 1D electron systems and present a generalization of the model of Rejec *et al* for the case of the spin $J > 1/2$ localized in the region of the QPC [16]. The physical realization of this hypothetical situation could be, for example, a QPC with an embedded Mn^{2+} ion with spin $J = 5/2$. Alternatively, it could correspond to a QPC containing several localized electrons in the regime of spontaneous spin polarization. The latter situation probably corresponds to the case of the long quantum wires experimentally studied in the work of Reilly *et al* [9]. As already mentioned, in this case the additional plateau is formed around $G \approx 0.5G_0$ rather than $G \approx 0.7G_0$.

Firstly, let us consider the situation qualitatively. Having non-zero magnetic moment J, the localized state affects the propagating carriers via exchange interaction of the Heisenberg type. The transmission coefficient through the QPC thus appears to be spin dependent. Indeed, after entry of the propagating electron into the QPC, its total spin can be either $S_1 = J + 1/2$ or $S_2 = J - 1/2$. The number of possible realizations of configuration 1 is $N_1 = 2S_1 + 1 = 2J + 2$ against $N_2 = 2S_2 + 1 = 2J$ for configuration 2. In the case of the ferromagnetic interaction the energy of state 1 lies below the energy of state 2, and thus the potential barrier formed in the region of the QPC is higher for configuration 2. Consequently, for small enough chemical potentials the ingoing electron in configuration 1 passes freely through the QPC while in configuration 2 it is reflected. Then, only configuration 1 contributes to the conductance. In the absence of the external magnetic field the probability of its realization is $(J + 1)/(2J + 1)$ and thus the conductance of the QPC in the considered regime reads

$$G_f = \frac{J+1}{2J+1}G_0. \tag{10}$$

[3] The possibility of localization of the carriers in the region of the QPC was recently analyzed in experimental work by Yoon *et al* [36].

Table 1. Possible values of the conductance for different values of J.

J	Ferromagnetic interaction, G_f	Antiferromagnetic interaction, G_a
1/2	$(3/4)G_0$	$(1/2)G_0$
1	$(2/3)G_0$	$(1/3)G_0$
3/2	$(5/8)G_0$	$(3/8)G_0$
2	$(3/5)G_0$	$(2/5)G_0$
5/2	$(7/12)G_0$	$(5/12)G_0$
∞	$(1/2)G_0$	$(1/2)G_0$

In contrast, in the case of the antiferromagnetic interaction, configuration 2 is energetically preferable, and the conductance should be

$$G_a = \frac{J}{2J+1} G_0. \tag{11}$$

Table 1 summarizes the possible values of the conductance for different values of J. In the case of the ferromagnetic interaction, which is likely to be realized in the experiment the height of the sub-step is seen to decrease with the number of impaired electrons localized on the contact, and reaches the value of $(1/2)G_0$ in the limit $J \to \infty$. Besides, the number of unpaired electrons could be expected to depend on the length of the QPC, being small for short and large for long contacts. Thus, for the short contacts one can expect the conductance to be about $0.7G_0$ while for the long wires it should attain the value of $0.5G_0$. This result corresponds perfectly to the experimental observations by Reilly *et al* [9, 18, 19]. The application of the external magnetic field leads to the spin polarization of both propagating and localized carriers, thus transforming the conductance into $G_0 = e^2/h$ for all values of J, as seen in the experiment.

It should be noted that, in contrast to [35], in our picture the coexistence of the two plateaux (e.g. $0.75G_0$ and $0.25G_0$) is forbidden. The difference comes from the difference of the effective potential seen by the propagating electron. In our case they represent single rectangular barriers with spin-dependent height, while in [35] they are considered as resonant double barrier structures with two quasilevels corresponding to singlet and triplet configurations split by exchange interaction.

Now, let us calculate the conductance in a more rigorous way. If the external magnetic field is absent and electrons in the ingoing and outgoing leads are unpolarized, the density matrix of the system containing the free propagating electron and the localized spin before their interaction reads

$$\boldsymbol{\rho}_{\text{in}} = \boldsymbol{\rho}_e \otimes \boldsymbol{\rho}_J = \tfrac{1}{2}(|{+1/2}\rangle\langle{+1/2}| + |{-1/2}\rangle\langle{-1/2}|) \otimes \left(\frac{1}{2J+1}\sum_{m=0}^{2J} |J-m\rangle\langle J-m|\right). \tag{12}$$

There are $4J + 2$ possible mutual orientations of the spin of the propagating and localized electrons. For each of them after passing the region of the QPC the spin of the propagating electron can be either conserved or inverted as a result of exchange interaction, while the total spin of the pair is conserved. The conductance at zero temperature can be thus calculated as

$$G(0,E) = \frac{e^2}{4h(2J+1)} \sum_{m=0}^{2J} [|A_{|-\frac{1}{2};J-m\rangle \to |-\frac{1}{2};J-m\rangle}|^2 + |A_{|-\frac{1}{2};J-m+1\rangle \to |\frac{1}{2};J-m\rangle}|^2 + |A_{|\frac{1}{2};J-m\rangle \to |\frac{1}{2};J-m\rangle}|^2 + |A_{|\frac{1}{2};J-m\rangle \to |-\frac{1}{2};J-m+1\rangle}|^2], \tag{13}$$

where A denotes the transmission amplitudes dependent on the Fermi energy of carriers. The indices of the transmission amplitudes denote the spin state of the propagating and localized electrons before and after interaction. Thus, the amplitudes $A_{|-1/2;J-m\rangle \longrightarrow |-1/2;J-m\rangle}$ and $A_{|1/2;J-m\rangle \longrightarrow |1/2;J-m\rangle}$ describe the spin-conservative passing of the carrier through the QPC, while $A_{|-1/2;J-m+1\rangle \longrightarrow |1/2;J-m\rangle}$ and $A_{|1/2;J-m\rangle \longrightarrow |-1/2;J-m+1\rangle}$ correspond to the passing accompanied by a spin flip.

To determine the values of the transmission amplitudes in equation (13), it is necessary to specify the Hamiltonian of the interaction between the propagating carrier and localized spin J. In the present work we suppose that they interact only in the region of length L (dimension of the QPC), whereas in the other space the interaction is taken to be absent. The model Hamiltonian can be thus represented in the following form:

$$H = \begin{cases} \dfrac{\hbar^2 k^2}{2m_{\text{eff}}}, & x < 0, \quad x > L \\ \dfrac{\hbar^2 k^2}{2m_{\text{eff}}} + V_{\text{dir}} + V_{\text{ex}} \boldsymbol{\sigma} \cdot \mathbf{J}, & x \in [0, L] \end{cases} \tag{14}$$

where m_{eff} is an effective mass of the electron, $V_{\text{dir}} > 0$ is a matrix element of the direct interaction and V_{ex} is a matrix element of the exchange interaction. For the ferromagnetic interaction $V_{\text{ex}} < 0$, while for the antiferromagnetic interaction $V_{\text{ex}} > 0$. To calculate the spin-dependent transmission amplitudes one can represent the Hamiltonian in the region of the QPC using the basis of the $4J + 2$ vectors

$$|\psi_1\rangle = |1/2;\, J\rangle; \quad \cdots \quad |\psi_{2m}\rangle = |-1/2;\, J-(m-1)\rangle;$$
$$|\psi_{2m+1}\rangle = |1/2;\, J-m\rangle, \quad \cdots \quad |\psi_{4J+2}\rangle = |-1/2;\, -J\rangle, \tag{15}$$

where $m = 1, \ldots, 2J$. Due to the total spin conservation the matrix of the Hamiltonian has a block-diagonal form

$$H_{lk} = V_l^{(1)} \delta_{lk} + V_l^{(2)} (\delta_{l,k+1} + \delta_{l+1,k}), \tag{16}$$

where the parameters $V_1^{(1,2)}$ read

$$V_1^{(1)} = V_{4J+2}^{(1)} = \frac{\hbar^2 k^2}{2m_{\text{eff}}} + V_{\text{dir}} + V_{\text{ex}} J,$$
$$V_1^{(2)} = V_{4N+2}^{(2)} = 0,$$
$$V_{2m}^{(1)} = \frac{\hbar^2 k^2}{2m_{\text{eff}}} + V_{\text{dir}} - V_{\text{ex}}(J-m+1),$$
$$V_{2m+1}^{(2)} = 0, \qquad V_{2m+1}^{(1)} = \frac{\hbar^2 k^2}{2m_{\text{eff}}} + V_{\text{dir}} + V_{\text{ex}}(J-m),$$
$$V_{2m}^{(2)} = V_{\text{ex}} \sqrt{m(2J-m+2)}. \tag{17}$$

This Hamiltonian can be reduced to the diagonal form $H_{lk} = \varepsilon_l \delta_{lk}$, where

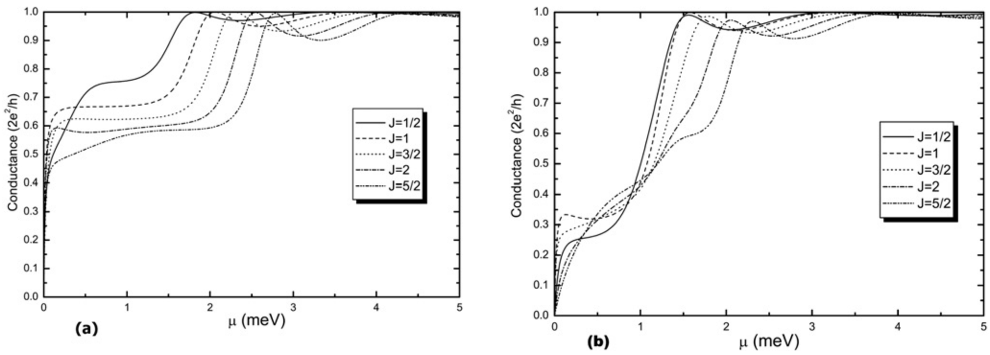

Figure 1. Dependence of the conductance of the QPC on the chemical potential for different values of the net localized spin J. The parameters of the calculation were taken as follows: the effective mass of the carrier $m_{\text{eff}} = 0.06m_e$, the temperature $T = 4$ K, the length of the contact $L = L_0 J$ with $L_0 = 50$ nm, $V_{\text{dir}} = 1$ meV, $|V_{\text{ex}}| = 0.48$ meV, for the (a) ferromagnetic and (b) antiferromagnetic couplings.

$$\varepsilon_{2m+1}\varepsilon_1 = \varepsilon_{4J+2} = \frac{\hbar^2 k^2}{2m_{\text{eff}}} + V_{\text{dir}} + V_{\text{ex}} J,$$
$$\varepsilon_2\varepsilon_2 = \frac{\hbar^2 k^2}{2m_{\text{eff}}} + V_{\text{dir}} - V_{\text{ex}}(J+1). \tag{18}$$

The value of ε_1 given by formula (18) corresponds to the total spin of the localized plus propagating electron that is equal to $S_1 = J + 1/2$, while the value of ε_2 corresponds to the total spin $S_2 = J - 1/2$.

To determine the transmission amplitudes, it is necessary to obtain the general expression for the wavefunction for all possible mutual orientations of the spin of the propagating electron and the localized spin. To give an example, let us consider the case when spin projection of an ingoing electron is $-1/2$ while the projection of the localized spin corresponds to $J - m + 1$. In this case one has

$$\Psi_{\text{I}}(x) = \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{ik_F x} + B_{|-\frac{1}{2};J-m+1\rangle\to|-\frac{1}{2};J-m+1\rangle} \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{-ik_F x}$$
$$+ \; B_{|-\frac{1}{2};J-m+1\rangle\to|\frac{1}{2};J-m\rangle} \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-ik_F x}, \qquad x < 0,$$
$$\Psi_{\text{II}}(x) = \mathbf{X}_m^{(1)}(C_{+1m}e^{ik_1 x} + C_{-1m}e^{-ik_1 x})$$
$$+ \; \mathbf{X}_m^{(2)}(C_{+2m}e^{ik_2 x} + C_{-2m}e^{-ik_2 x}), \qquad x \in [0, L], \tag{19}$$
$$\Psi_{\text{III}}(x) = A_{|-\frac{1}{2};J-m+1\rangle\to|-\frac{1}{2};J-m+1\rangle} \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{ik_F x}$$
$$+ \; A_{|-\frac{1}{2};J-m+1\rangle\to|\frac{1}{2};J-m\rangle} \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{ik_F x}, \qquad x > L.$$

where the indices I, II and III denote left lead, QPC and right lead respectively, A and B are transmission and reflection amplitudes, k_F is the Fermi wavenumber of the carrier inside the leads, the wavenumbers k_1 and k_2 correspond to the eigenenergies of the carrier in the region with exchange interaction (see formulas (18))

$$k_2 = \sqrt{\frac{2m_{\text{eff}}}{\hbar^2}[E_F - V_{\text{dir}} - V_{\text{ex}}(J+1)]},$$
$$k_1 = \sqrt{\frac{2m_{\text{eff}}}{\hbar^2}[E_F - V_{\text{dir}} + V_{\text{ex}} J]}, \tag{20}$$

and $\mathbf{X}_m^{(1,2)}$ are eigenvectors of the mth block of the Hamiltonian (16)

$$\mathbf{X}_m^{(1)} = \frac{1}{\sqrt{2J+1}} \begin{pmatrix} \sqrt{2J-m+1} \\ -\sqrt{m} \end{pmatrix},$$
$$\mathbf{X}_m^{(2)} = \frac{1}{\sqrt{2J+1}} \begin{pmatrix} \sqrt{m} \\ \sqrt{2J-m+1} \end{pmatrix}. \tag{21}$$

Expressions (18)–(20) together with the continuity condition for the wavefunction and its derivative in the points $x = 0$ and L allow the determination of all the transmission amplitudes present in the formula for the conductance, equation (13).

Figure 1 shows the dependence of the conductance of the QPC on the chemical potential for different values of the net localized spin J. One clearly sees the formation of the plateaux of the ballistic conductance different from $0.75G_0$ for the case $J > 1/2$ in accordance with formulas (10) and (11).

Similar to the case of the single localized electron considered in section 2, the many-electron correlations should lead to the temperature-dependent renormalization of V_{ex} and quenching of the fractional plateaux for small temperatures.

3. Fractional quantization of ballistic conductance in 1D hole systems

As already mentioned in section 1, fractional quantization of the ballistic conductance can be qualitatively different in the samples with n- and p-type conductivity. The reason is a different spin structure of the electrons and holes in such semiconductor materials as Si, Ge and GaAs. The valence bands of these bulk semiconductors consist of a heavy hole band with spins $J_z^{\text{hh}} = \pm 3/2$, and a light hole band with spins $J_z^{\text{lh}} = \pm 1/2$.[4] In low dimension due to the effects of confinement the energetic splitting Δ appears between these two bands. It depends on the width of the quantum wire, the difference of the effective masses of the light and heavy holes, m_{lh} and m_{hh}, and strains.

The spin-dependent scattering of the localized and freely propagating holes can be considered in a similar way as

[4] We neglect here the third band, which is split off by the exchange interaction from the bands of the heavy and light holes for $k = 0$.

considered for the electrons in the previous section. We consider the transmission of freely propagating hole states facing effective potential barriers generated by a spin-dependent interaction, $V_{\rm dir} + V_{\rm ex}\mathbf{J}_{\rm p} \cdot \mathbf{J}_{\rm l}$, with a localized hole supposedly present in the region of the QPC. The indices p and l correspond to the propagating and localized holes respectively, $V_{\rm dir} > 0$ is the matrix element of the direct interaction and $V_{\rm ex}$ is the matrix element of the exchange interaction.

Let us first qualitatively understand how the difference of the spin structure of electrons and holes is reflected in the patterns of fractional quantization of the ballistic conductance for them. The spin-dependent part of the effective Hamiltonian for the holes can be recast as $V_{\rm ex}\mathbf{J}_{\rm p}\cdot\mathbf{J}_{\rm l} = V_{\rm ex}(\mathbf{J}_{\rm T}^2-\mathbf{J}_{\rm p}^2-\mathbf{J}_{\rm l}^2)/2 = V_{\rm ex}[J_{\rm T}(J_{\rm T}+1) - 3(3/2+1)]/2$. Thus, for the possible absolute values of the total spin of the hole pair $J_{\rm T} = 3, 2, 1, 0$ one has the following values of the heights of the spin-dependent effective potential barriers: $V_{\rm dir} + 9V_{\rm ex}/4$, $V_{\rm dir} - 3V_{\rm ex}/4$, $V_{\rm dir} - 11V_{\rm ex}/4$ and $V_{\rm dir} - 15V_{\rm ex}/4$, respectively. In the case of the ferromagnetic interaction the barrier is lowest for the largest possible spin, while in the case of the antiferromagnetic interaction the opposite situation holds. Now, for an unpolarized initial state ($B = 0$), the probabilities of realization of the $J_{\rm T} = 3, 2, 1, 0$ configurations of the total spin are 7/16, 5/16, 3/16 and 1/16. Therefore, considering the hypothetical case of the zero offset between light and heavy hole bands, $\Delta = 0$,[5] in the ferromagnetic case we expect to obtain plateaux close to the values $(7/16)G_0 = 7e^2/4h$, $(12/16)G_0 = 3e^2/h$ and $(15/16)G_0 = 15e^2/4h$ (given that the conductance of the noninteracting system is quantized in the units $4e^2/h$ in this case). In the antiferromagnetic case plateaux close to $e^2/4h$, e^2/h and $9e^2/4h$ are expected.

The situation can be qualitatively different if the offset between the bands of light and heavy holes is not negligible. For example, in the limit $\Delta \to \infty$ only the heavy hole band is available for both localized and propagating carriers. The conductance of the noninteracting system is thus quantized in the units $2e^2/h$ as for the case of the electrons. The spins of the pair of holes can be either parallel or antiparallel with equal probability of realization. Due to the huge offset between light and heavy holes the spin-flip processes are blocked[6] and we expect just one additional plateau in the fractional conductance corresponding to $G = e^2/h$. The situation is thus different from the case of electrons, where spin-flip processes are allowed and result in an additional plateau at $G = 3e^2/2h$ [32, 33, 35].

Clearly, in the realistic case of finite Δ the situation goes beyond the two extreme cases considered above. To account for it a more detailed analysis of the model is needed. As for the case of the electrons we suppose that the propagating and localized holes interact only in the region of the QPC having length L. The Hamiltonian of the system can be thus cast in the form

$$H = \begin{cases} \dfrac{\hbar^2 k^2}{2m_{\rm hh,lh}}, & x < 0, \quad x > L \\ \dfrac{\hbar^2 k^2}{2m_{\rm hh,lh}} + V_{\rm dir} + V_{\rm ex}\mathbf{J}_{\rm p}\cdot\mathbf{J}_{\rm l}, & x \in [0, L]. \end{cases} \tag{22}$$

The initial state of the system before the interaction can be represented by the following density matrix:

$$\boldsymbol{\rho}_{\rm in} = \boldsymbol{\rho}_{\rm p} \otimes \boldsymbol{\rho}_{\rm l}, \tag{23}$$

$$\boldsymbol{\rho}_{\rm p,l} = \sum_{\rm p,l=\pm 3/2,\pm 1/2} \alpha_{m_{\rm p,l}}(T, E)|m_{\rm p,l}\rangle\langle m_{\rm p,l}|, \tag{24}$$

where $\boldsymbol{\rho}_{\rm p}$ and $\boldsymbol{\rho}_{\rm l}$ are the density matrices associated with the free propagating and localized hole states respectively. In the absence of an external magnetic field we suppose that the propagating and localized holes are unpolarized, and at $T = 0$ K one has $\alpha_{\pm 3/2_{\rm p}}(0, E) = \alpha_{\pm 3/2_{\rm l}}(0, E) = \theta(E - E_{\rm F})/N$, $\alpha_{\pm 1/2_{\rm p}}(0, E) = \alpha_{\pm 1/2_{\rm l}}(0, E) = \theta(E - \Delta - E_{\rm F})/N$, where N denotes the number of open propagating channels ($N = 2$ if $E_{\rm F} \leqslant E < E_{\rm F} + \Delta$ and $N = 4$ if $E \geqslant E_{\rm F} + \Delta$).

In the $\{|m_{\rm p}, m_{\rm l}\rangle\}$ basis the Hamiltonian in the region of the QPC can be represented by a block-diagonal 16×16 matrix

$$H = {\rm diag}[H^{(+3)}; H^{(+2)}; H^{(+1)}; H^{(0)}; H^{(-1)}; H^{(-2)}; H^{(-3)}], \tag{25}$$

with each block associated with a given (superscript) value of the z-component of the total angular momentum of the pair of holes, $J_{T,z} = m_{\rm p} + m_{\rm l}$, having dimension equal to $4 - |J_{T,z}|$ and presenting the following matrix elements:

$$\begin{gathered} H^{(\pm 3)} = E^0_{\rm hh} + \tfrac{9}{4}V_{\rm ex}, \qquad H^{(\pm 2)}_{11} = E^0_{\rm hh} + \tfrac{3}{4}V_{\rm ex} + \Delta, \\ H^{(\pm 2)}_{22} = E^0_{\rm lh} + \tfrac{3}{4}V_{\rm ex} + \Delta, \qquad H^{(\pm 2)}_{12} = \tfrac{3}{2}V_{\rm ex}, \\ H^{(\pm 1)}_{11} = E^0_{\rm hh} - \tfrac{3}{4}V_{\rm ex} + \Delta, \qquad H^{(\pm 1)}_{22} = E^0_{\rm lh} + \tfrac{1}{4}V_{\rm ex} + 2\Delta, \\ H^{(\pm 1)}_{33} = E^0_{\rm lh} - \tfrac{3}{4}V_{\rm ex} + \Delta, \qquad H^{(\pm 1)}_{12} = H^{(\pm 1)}_{23} = V_{\rm ex}\sqrt{3}, \\ H^{(0)}_{11} = H^{(0)}_{44} = E^0_{\rm hh} - \tfrac{9}{4}V_{\rm ex}, \\ H^{(0)}_{22} = H^{(0)}_{33} = E^0_{\rm lh} - \tfrac{1}{4}V_{\rm ex} + 2\Delta, \\ H^{(0)}_{12} = H^{(0)}_{34} = \tfrac{3}{2}V_{\rm ex}, \qquad H^{(0)}_{23} = 2V_{\rm ex}, \end{gathered} \tag{26}$$

with $H_{ij} = H_{ji}$, $E^0_{\rm hh,lh} = \frac{\hbar^2 k^2}{2m_{\rm hh,lh}} + V_{\rm dir}$ and the other matrix elements equal to zero.

The general expression for the conductance of the system at zero temperature is given by

$$\begin{aligned} G(0, E_{\rm F}) = \frac{Ne^2}{h} &\sum_{m_{\rm p},m_{\rm l},m'_{\rm p},m'_{\rm l}=\pm 3/2,\pm 1/2} \alpha_{m_{\rm p}}(0, E_{\rm F})\alpha_{m_{\rm l}}(0, E_{\rm F}) \\ &\times |A(E_{\rm F})_{m_{\rm p},m_{\rm l}\to m'_{\rm p},m'_{\rm l}}|^2 \delta_{m_{\rm p}+m_{\rm l},m'_{\rm p}+m'_{\rm l}}. \end{aligned} \tag{27}$$

The transmission amplitudes, $A_{m_{\rm p},m_{\rm l}\to m'_{\rm p},m'_{\rm l}}$, are determined by finding the stationary states of the corresponding propagating hole facing the effective potential barrier described by the

[5] It should be noted that the second level of heavy holes 2hh lies usually very close to the first level of the light holes 1lh and in general cannot be neglected. However, the presence of the strains usually leads to the lowering of the 1lh, and thus the situation when it lies closer to 1hh than to 2hh can be realized (see [50]).

[6] Indeed, the spin-flip process of the type $+3/2, -3/2 \to -3/2, +3/2$ always involves light holes as intermediate states, $+3/2, -3/2 \to +1/2, -1/2 \to -1/2, +1/2 \to -3/2, +3/2$, and its intensity thus goes to zero for large offsets Δ.

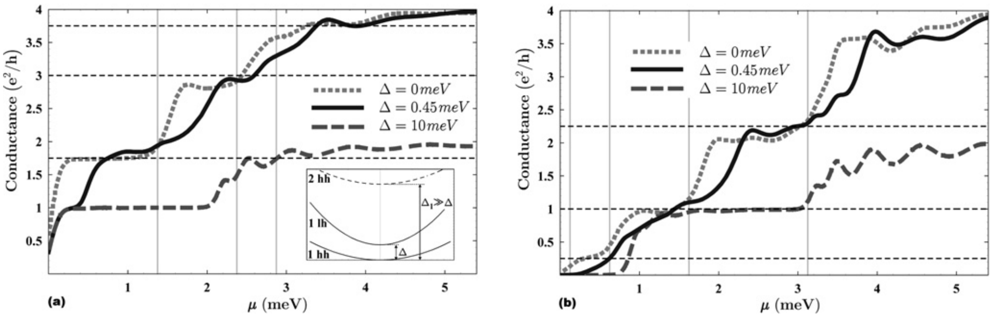

Figure 2. (a) Steps of the quantum conductance staircase versus the chemical potential of carriers in a GaAs QPC for the case of ferromagnetic exchange interaction between localized and propagating holes. The different lines correspond to three different values of $\Delta(\text{meV}) = 0, 0.45, 10$. The direct and exchange interaction are estimated as $V_{\text{ex}} \simeq -0.5$ meV and $V_{\text{dir}} \simeq 1$ meV, the length of the contact as $L = 65$ nm and we considered the temperature $T = 0.5$ K. The vertical gray lines correspond to the values of the heights of the effective potential barriers, $V_{\text{dir}} - 3V_{\text{ex}}/4$, $V_{\text{dir}} - 11V_{\text{ex}}/4$ and $V_{\text{dir}} - 15V_{\text{ex}}/4$, whereas the dashed horizontal lines correspond to the values $7e^2/4h$, $3e^2/h$ and $15e^2/4h$. The inset shows a band structure of the 1D holes. (b) The same for antiferromagnetic interaction, $V_{\text{ex}} \simeq 0.5$ meV, and supposing $V_{\text{dir}} \simeq 2$ meV. The dashed horizontal lines correspond to the values $e^2/4h$, e^2/h and $9e^2/4h$.

above $H^{(J_{T,z})}$ matrices. Considering the conservation of the total spin represented by the Kronecker δ one gets 44 different transmission amplitudes. In the absence of an external magnetic field this number can be reduced to 22 due to the spin inversion invariance.

The transmission amplitudes associated with the initial states $|+\frac{3}{2}, +\frac{3}{2}\rangle$ and $|-\frac{3}{2}, -\frac{3}{2}\rangle$, corresponding to the values $J_{T,z} = \pm 3$, are spin conservative and thus determined by solving the problem of a free particle with kinetic energy $\hbar^2 k_{\text{F}}^2/2m_{\text{hh}}$ facing an effective rectangular barrier of width L and height $V_{\text{dir}} + \frac{9}{4}V_{\text{ex}}$. The first non-trivial spin-flip processes are associated with the states with $J_{T,z} = \pm 2$. The transmission amplitudes in this case, as well as in the cases of $J_{T,z} = \pm 2, \pm 1, 0$, can be determined from a procedure analogical to one described earlier for the case of the electrons.

Figures 2(a) and (b) show the conductance of a GaAs-based QPC for various offsets between the bands of light and heavy holes. For moderate values of Δ, we observe the expected plateaux close to the values $7e^2/4h$, $3e^2/h$ and $15e^2/4h$ for the ferromagnetic exchange interaction and the plateaux close to $e^2/4h$, e^2/h and $9e^2/4h$ for the antiferromagnetic exchange interaction, together with the additional plateau e^2/h for the ferromagnetic case. It should be noted that the quantization of the ballistic conductance associated with holes in Si and Ge structures is expected to be qualitatively the same as in GaAs because of the similarity of the spin structure of the valence band in these materials. However, it can be qualitatively different in IV–VI semiconductors such as PbTe, PbSe and PbS, where the electron–hole symmetry holds.

In addition, we also analyzed the effects of an applied external magnetic field parallel to the structure growth axis which produces the Zeeman splitting of the light and heavy hole bands, $H^{\text{lh,hh}} = -g_{\parallel}^{\text{lh,hh}} \mu_b J_z^{\text{lh,hh}} B$, where μ_{b} is the Bohr magneton and $g_{\parallel}^{\text{lh,hh}}$ are the parallel components of the effective g-factor tensors of the light and heavy hole subbands. In our calculations for GaAs QPCs, we express the g-factors as $g_{\parallel}^{\text{lh}} = 2\kappa$ and $g_{\parallel}^{\text{hh}} = 6\kappa$, with the Luttinger parameter

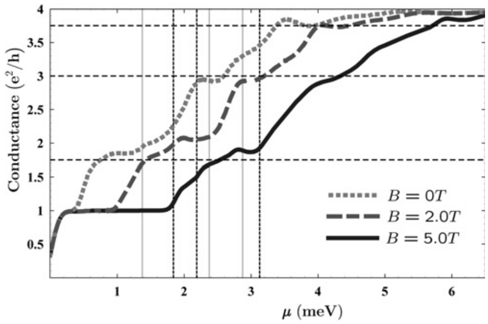

Figure 3. Effect of an external magnetic field applied along the growth axis of a GaAs QPC. Except for the applied magnetic field, we used the same parameters as figure 2, with $\Delta = 0.45$ meV. We see that moderate magnetic fields, $B = 2$ and 5 T, reduce the relative width of the steps and change their position. The values of the effective potential barriers appear as vertical gray lines, while the energies of the bottom of the subbands, for $B = 5$ T, appear as vertical dotted lines. Observe that there is an inversion between the light and heavy hole subbands because of the difference in their effective g-factor and the moderate magnetic field.

estimated as $\kappa \sim -0.6$.[7] So, the propagating and localized light and heavy hole subbands will be split with the energies $\Delta_Z^{\text{lh}} = -2\kappa\mu_b B$ and $\Delta_Z^{\text{hh}} = -18\kappa\mu_{\text{b}} B$, respectively. In this case we do not have invariance with respect to the spin inversion and the conductance of the system will be given by the sum of all 44 distinct transmission amplitudes present in equation (25). The results of our calculation are summarized in figure 3. The Zeeman splitting in each of the light and heavy hole subbands, in addition to the Δ splitting between them, increases the energy dependence of the initial state's probabilities of realization and of the scattering processes. Therefore, under a moderate magnetic field, the first plateau at e^2/h shows an increase in its width, till the bottom energy of the second band, while the other ones have their relative widths reduced and their energy changed. This behavior is illustrated

[7] In bulk GaAs $\kappa \sim -1.2$. Quantum wells with widths of order $L \sim 30$ nm present Luttinger parameter $\kappa \sim -0.3$, see [51]. So, we estimate an intermediate value for κ in the QPC that we assume.

by the plotted conductances for $B = 2$ and 5 T in figure 3. However, in systems under a stronger external magnetic field or presenting bigger g-factors, such that the heights of the spin effective potential barrier are smaller than the splitting between the different subbands, the conductance of the system tends to show plateaux at the integer values of e^2/h [52].

4. Conclusions

In conclusion, we have shown that the presence of the uncompensated spin J in the region of the quantum point contact can result in the fractional quantization of the ballistic conductance. For samples with n-type conductivity single localized electrons with spin $J = 1/2$ result in the appearance of the single additional plateau with $G = 0.75G_0$. The increase of J with increase of the length of the contact results in the decrease of the fractional conductance to $0.5G_0$. In the samples with p-type conductivity the situation is much more complicated due to the complex spin structure of the valence band in the materials like Si, Ge and GaAs. Depending on the offset between the bands of the light and heavy holes one or several fractional plateaux can be observed in this case.

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